ABBOTT

Theoretical Estimation Of the Internal Speed Variation of a 3500 Horse-Power Vertical Cross-Compound Engine

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THEORETICAL ESTIMATION OF THE INTERNAL SPEED VARIATION OF A 3500 HORSE-POWER VERTICAL CROSS-COMPOUND ENGINE.

...BY...

William Lamont Abbott

THESIS

FOR THE DEGREE OF MECHANICAL ENGINEER

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

WILLIAM LAMONT ABBOTT

ENTITLED THEORETICAL ESTIMATION OF THE INTERNAL SPEED VARIATION

OF A 3500 HORSE-POWER VERTICAL CROSS-COMPOUND ENGINE

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE

OF Mechanical Engineer

L. P. Breckenridge

HEAD OF DEPARTMENT OF Mechanical Engineering.

THEORETICAL ESTIMATION OF THE INTERNAL SPEED VARIATION OF A 3500 HORSE-POWER VERTICAL CROSS-COMPOUND ENGINE.

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The object of the investigation forming the subject of this paper, was to determine the amount of the angular variation which would occur during a revolution, in the relative positions of the armatures of alternating current dynamos, if connected to two similiar engines of the type described below, when running in parallel at the same number of revolutions per minute; and to see if the variation in phase would be great enough to put the dynamos out of step, or if the amount of cross current would be objectionable.

The engine to which the calculations apply is a vertical cross-compound Corliss, 30 X 80 X 60, condensing engine, built by the E.P. Allis Company, of Milwaukee, Wisconsin, and is to be directly connected to one ACH 40-2500-75 double-current General Electric generator for use at the Harrison Street power-house of the Chicago Edison Company.

The condition of the load under which the engine was assumed to be running was that of the economical rating, or 3500 I.H.P., with a constant speed of 75 revolutions per minute.

The indicator cards from which the determinations were made, were derived from a set of cards obtained from the Allis Company taken on Nov.28,1898, from the Allis Corliss cross-compound engine No.4, of the South Side Elevated Railway Company

of this city, while the engine was running under a steam pressure of 150 pounds per square inch. The cards were used merely to get the steam distribution of a Corliss engine working at a moderate load, and the ordinates of the cards were increased to 170 pounds per square inch, the latter being the pressure at which the new engine is designed to operate.

The various steps of the investigation in their proper order may be briefly enumerated as follows:

1. From the known weights and velocities of the reciprocating parts, calculate the force due to inertia, as modified by the length of the connecting rod, and construct a curve showing the same for all positions of the piston.

2. Obtain a pair of indicator diagrams (head and crank end) from each cylinder and deduct from the ordinates of each the simultaneous back pressure shown on the diagram taken from the opposite end of the cylinder.

3. Multiply the ordinates of these combined diagrams by the number of square inches of piston area, and plot a new curve showing the total pressures on the piston at each point of the stroke.

4. Combine the pressures shown by this last curve with those of the inertia curve to get a curve showing the total pressures on the crosshead.

5. Resolve the pressures represented by the ordinates of the combined steam-pressures and inertia curve, to determine the rotative effect at the crank-pin, and plot a curve showing the tangential effort on the pin for all positions of the piston.

6. Combine the curves of tangential effort for each cylinder to obtain a curve of crank effort for all cranks.

7. Draw a line through this last curve to represent the mean effective pressure of the varying forces acting during one revolution. Now, disregarding the original base-line, assume the forces above and below this mean line as positive and negative respectively, and from these positive and negative forces calculate the acceleration and retardation which would be produced upon the rotating mass connected to the engine shaft. With ordinates obtained in this way plot a curve showing the resultant velocities attained during each interval of the stroke.

8. From this curve of velocities, whose integrated sum must be zero, calculate the distance traveled by the crank-pin ahead of or behind some other point assumed to be revolving with an absolutely uniform motion at the same number of revolutions per minute, and plot a curve showing the amount of variation in the position of the pin, from mornal, at the different points of the stroke. The maximum ordinate of this curve in degrees of arc or degrees of electrical phase, is the result we are seeking.

A more detailed description of the various methods employed in the above steps follows:

TABLE I.

WEIGHT IN POUNDS OF RECIPROCATING PARTS.

Crosshead	----------6963	lb.
Connecting-rod (total 6500); one-half reciprocating	-------3250	"
Reciprocating weight - common to all cylinders	-----------10213	"
H.P. Piston = 2120; rod = 1466.	2120 + 1466 + 10213 =13799	"
L.P. Piston = 8040; rod = 2042.	8040 + 2042 + 10213 =20295	"

TABLE II.

SYMBOLS AND CONSTANTS.

Stroke --5 feet

R.P.M.--75

$V = \frac{5 \pi 75}{60}$ --19.63

V^2 --385.34

R (of crank) = --2.5 ft.

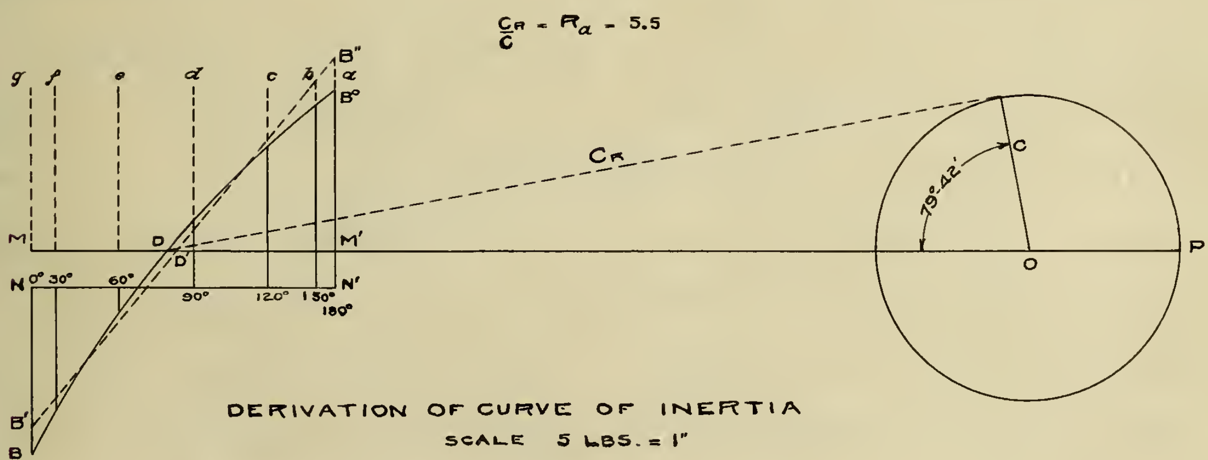
W = weight of reciprocating parts on one crank .

F_o = inertia of reciprocating parts at dead centre(connecting rod infinite) = $\frac{W}{g} \times \frac{V^2}{R}$ = ------------------------4.79 W

Crank --39 in.

Connecting rod --165 "

$\frac{\text{Crank}}{\text{Connecting rod}}$ ------------------------------------ $\frac{1}{5.5}$



DERIVATION OF CURVE OF INERTIA
SCALE 5 LBS. = 1″

Referring to Fig.1, draw line MP, representing in extent the combined length of the connecting rod and the piston travel. From the point O with radius OP strike a circle representing the crank travel. From the other end of the line MP lay off the distance MM', representing the piston travel. Now, assuming that a weight W equal to that of the reciprocating parts were concentrated at the centre of the crank pin, and were made to travel in the

crank-pin circle at the same number of revolutions as the engine is to run, their centrifugal force calculated from the formula $\frac{W v^2}{gR}$ would be, in this case, 4.79 X W. If again we assume that instead of traveling in the crank-pin circle, this weight were attached to the end of a connecting rod of infinite length, this formula would still represent the momentum, or inertia, at the instant of passing the dead centres. This inertia force would, of course, act in opposite directions at the opposite ends of the piston travel, and would be zero at the middle of the stroke. We, therefore, have three points of the inertia curve for a connecting rod of infinite length. Referring again to Fig.1, lay off at right angles to and below the line MM' the distance MB', representing the inertia at one end of the stroke, and the equal distance M'B" above the line, representing the inertia at the other end of the stroke, and draw a straight line, which will pass through the centre of the piston travel D' connecting these three points. This line B'D'B" represents the inertia curve for the reciprocating parts with the connecting rod of infinite length. To correct this curve for a rod 5½ cranks in length, lay off on line M'B" the distance B"B°, representing $\frac{1}{5.5}$ of the distance M'B". Again, on the line MB' produced, lay off B'B, which is also $\frac{1}{5.5}$ of the length of the line MB'. These points B and B° are two points on the curve of inertia for a connecting rod of 5½ cranks in length. To find the third point draw a line of the length of the connecting rod tangent to the crank-pin circle, and where this intersects the line MM' will be the third point on this inertia curve. Now strike the arc of a circle through the three points B,D,B°, and this will be the inertia curve for a connecting rod 5½ cranks long; and the ordinates in-

cluded between the line MM' and this curve represent the pressures upon the crosshead pin at the different points of the stroke . The magnitude of the resultant forces at various points of the stroke due to the inertia of the reciprocating weights are given in Table III.

TABLE III.

		High Pressure	Low Pressure
	Weight	13,799	20,295
Ordinate a = inertia at 180°	a ----------	69,000	101,475
" b = " 150°	b ----------	63,480	93,357
" c = " 120°	c ---------	49,680	73,062
" d = " 90°	d ---------	24,840	36,531
" e = " 60°	e ---------	-12,420	18,265
" f = " 30°	f ---------	-48,300	71,032
" g = " 0°	g ---------	-64,860	95,386

The engine under consideration being of the vertical type, lay off the line NN' below the line MM' at a distance representing the weight of the reciprocating parts. Ordinates then measured above or below NN' represent the pressures upon the crosshead pin due to both the inertia and the weight of the reciprocating parts.

Take now a pair of indicator diagrams,as shown in Fig.2 , and combine them so as to eliminate the back pressures, producing a curve representing the net pressure per square inch upon the piston. This curve I do not show, but instead I show a similiar curve A in Figs. 4 and 5, obtained by multiplying the ordinates of the curve not shown by the number of square inches in the piston (correcting for area taken out by the piston rod), and this curve represents the entire pressure upon the crosshead pin, due to steam pressure, upon a scale of 100,000 pounds per inch. Now draw in

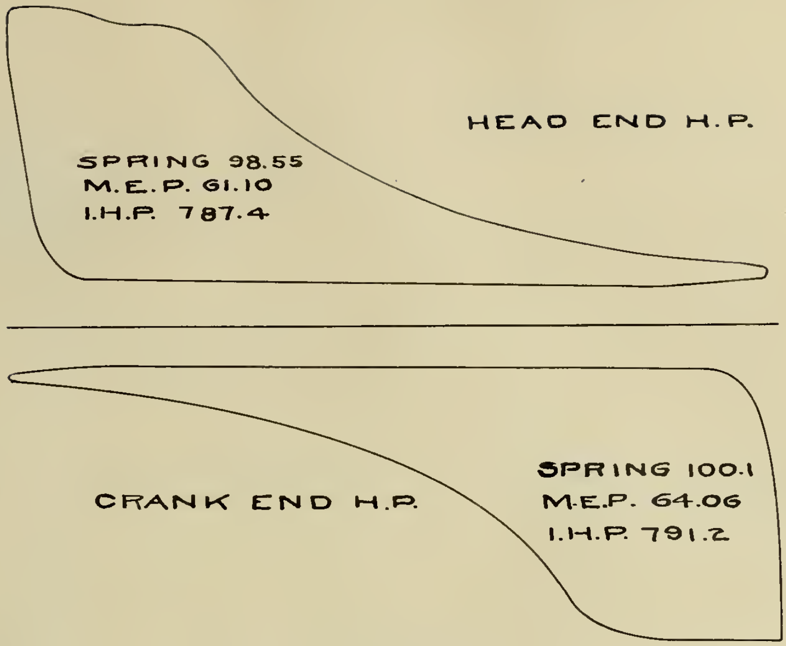

HEAD END H.P.
SPRING 98.55
M.E.P. 61.10
I.H.P. 787.4
CRANK END H.P.
SPRING 100.1
M.E.P. 64.06
I.H.P. 791.2

FIG. 2.

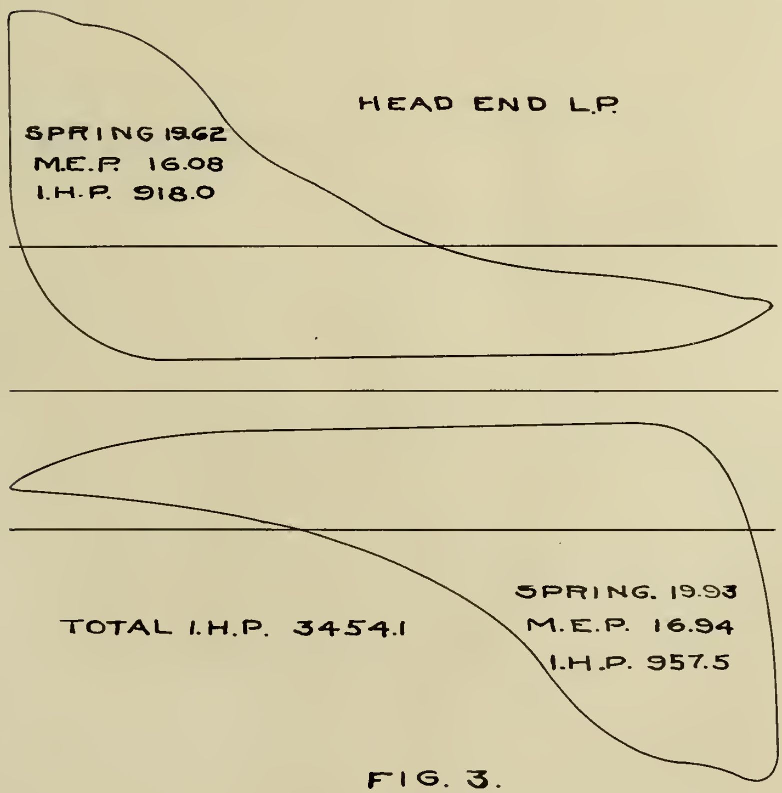

HEAD END L.P.
SPRING 19.62
M.E.P. 16.08
I.H.P. 918.0
TOTAL I.H.P. 3454.1
SPRING. 19.93
M.E.P. 16.94
I.H.P. 957.5

FIG. 3.

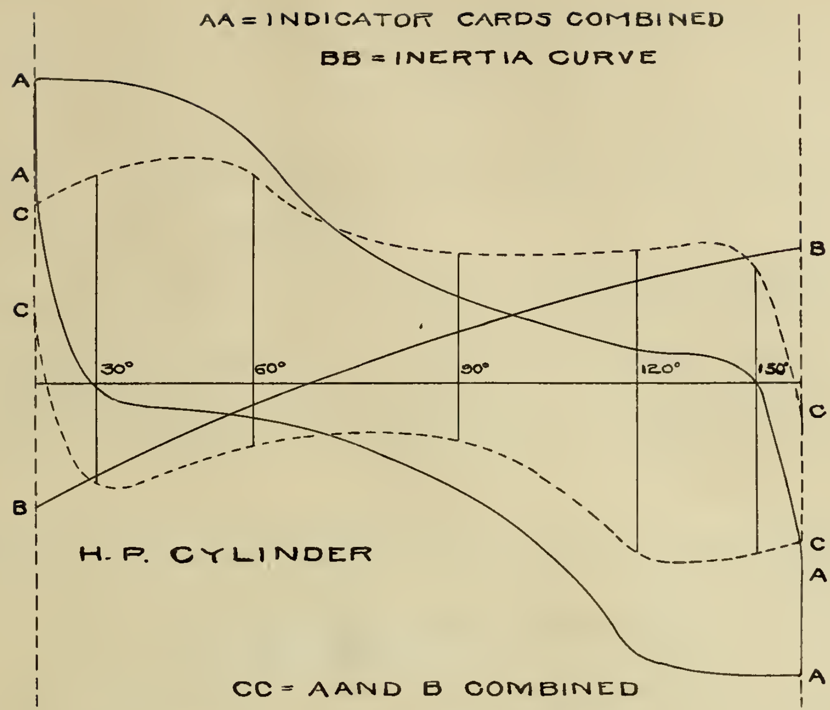

AA = INDICATOR CARDS COMBINED
BB = INERTIA CURVE
A
A
C
C
B
30°
60°
90°
120°
150°
B
C
C
A
A
H. P. CYLINDER
CC = A AND B COMBINED

FIG. 4.

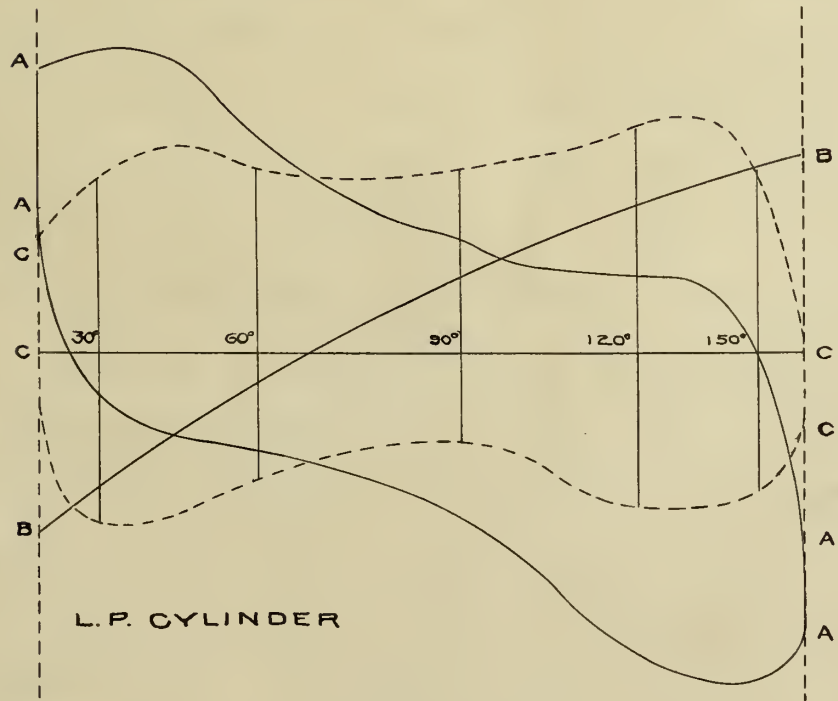

A
A
C
C
B
30°
60°
90°
120°
150°
B
C
C
A
A
L. P. CYLINDER

FIG. 5.

the inertia curve B to the same scale, and combine the curve A with the curve B, producing the curve C, all in Fig.4, and this last curve will represent the total pressure upon the crosshead pin due to the effects of steam and weight and inertia of the reciprocating parts. A summary of the data from which the inertia curve was deduced is given in Table I.

Thus far we have dealt with pressures referred to different points of the piston travel. We will now determine the effect of these different pressures at corresponding points of the crank-pin circle.

Extend the base-line of these curves a distance equal to the length of the connecting rod, and upon it strike the arc of a semicircle, not shown in the cut, representing the crank-pin travel; the centre of this semicircle being the length of the connecting rod from the middle of the line representing the piston travel. Divide this semicircle into six equal parts of 30 degrees each, and from these points of division as centres, with radius equal to the length of the connecting rod, describe arcs intersecting the base-line representing the piston travel. From these points of intersection erect ordinates perpendicular to the base-line, and where they intersect the curve C above and below the base-line will be twelve points, referred to piston travel, corresponding to twelve equidistant points in the crank-pin circle, for which we will compute the corresponding tangential pressures by the following method:

Referring now to Fig.6, upon the base-line AB strike a circle with a radius of unity; divide the circumference of this circle into twelve parts of 30 degrees each; through the points of division draw lines of the length of the connecting rod terminating

at the line AB. Produce these lines, if necessary, until they intersect the vertical line OC at the points e,e',etc. The distances Oe,Oe', etc., measured on the scale of which the radius OC is unity, represents the factors by which the pressures shown on the corresponding points of the curve C, Fig.4, should be multiplied in order to find the corresponding tangential pressures upon the crank pin. Determine in the same way the pressures corresponding to the twelve points selected, and, upon the base-line DD, Fig.7, representing to some convenient scale the crank-pin circle rectified, construct the curve as shown, the ordinates of which will represent the total rotative pressures for the upward and downward strokes. Determine by the same process the tangential pressure upon the crank pin of the low-pressure cylinder, as shown in Figs. 3 and 5, and construct this curve upon the same base-line, having corresponding points at a distance ahead of or behind the curve, for the high pressure cylinder depending upon the angle at which the cranks are set relatively to each other. This curve for the low-pressure cylinder is shown in Fig.7 by the dotted line. The combined effect of these two curves is shown in Fig.8, which represents the total crank effort for both cranks at all points during the revolution. Through this curve draw a line MM parallel to the base-line OO, in such a position that the areas included between the curve and the line MM shall be equal above and below. The position of the line MM may be conveniently determined by measuring with a planimeter the area included between the curve and the base-line OO, and dividing same by the length of the line OO.

We will now consider the equivalent mass of the rotating parts of the engine concentrated at the centre of the crank-pin, and as having no other velocity than that produced by the positive

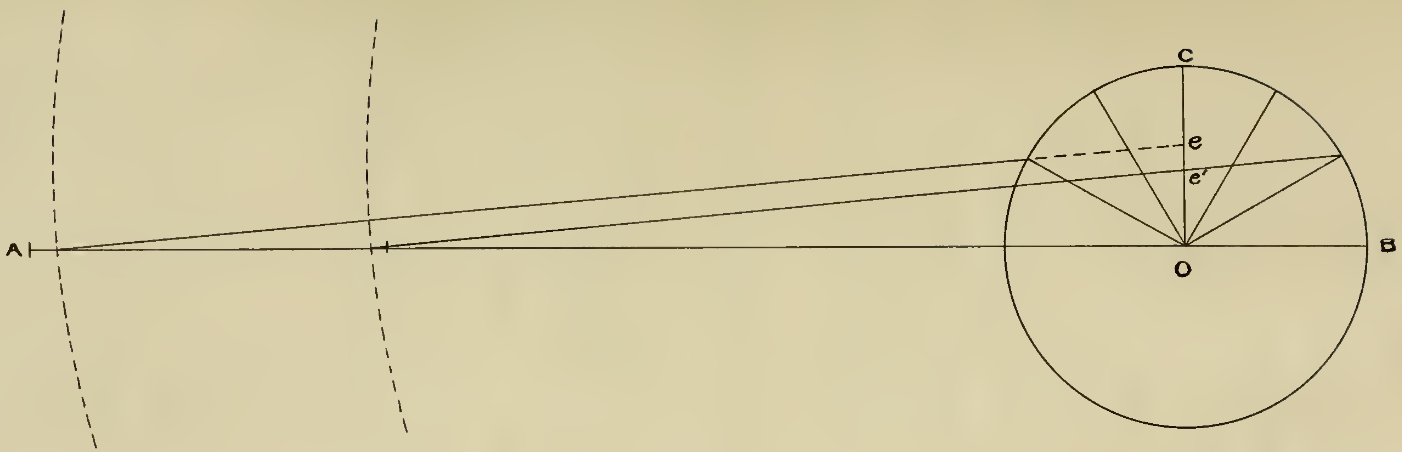
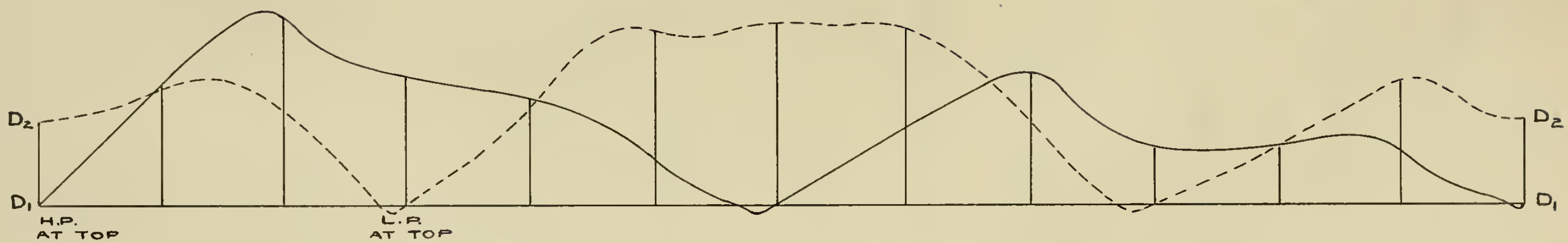


FIG. 6.



D_1 = ROTATIVE EFFECT OF H.P. CYLINDER
D_2 = " " L.P. "

FIG. 7.

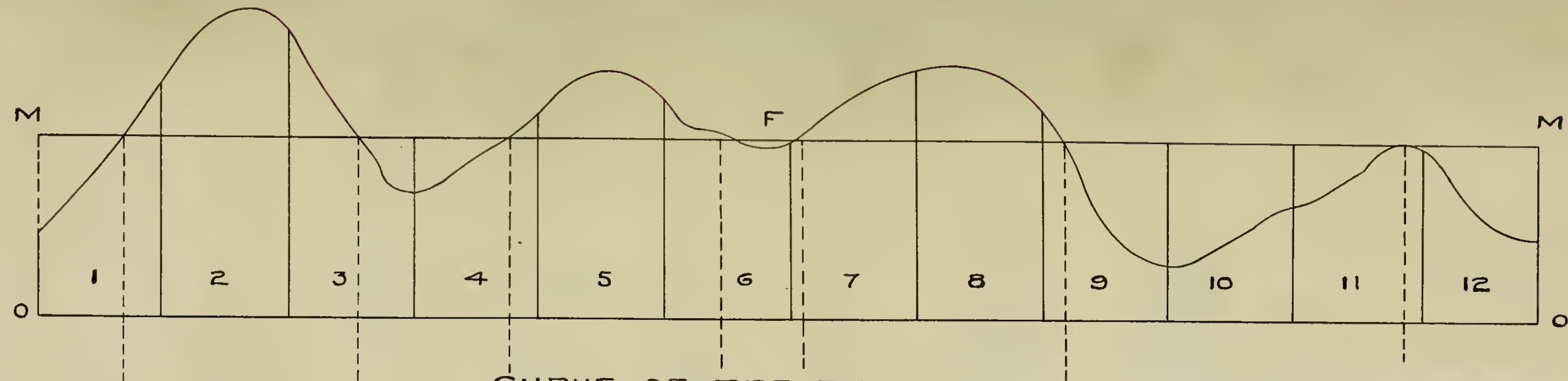
and negative forces represented by those portions of the curve of crank effort on either side of the line MM. For convenience in estimating we will assume that the applied force is uniform within each of the twelve spaces; i.e., this tangential force for each space, expressed in pounds above or below the normal MM, is equal to the mean height of each space above or below the line MM, and is exhibited in column F of Table IV.

The velocity gained or lost during each twelfth of a revolution is deduced as follows:

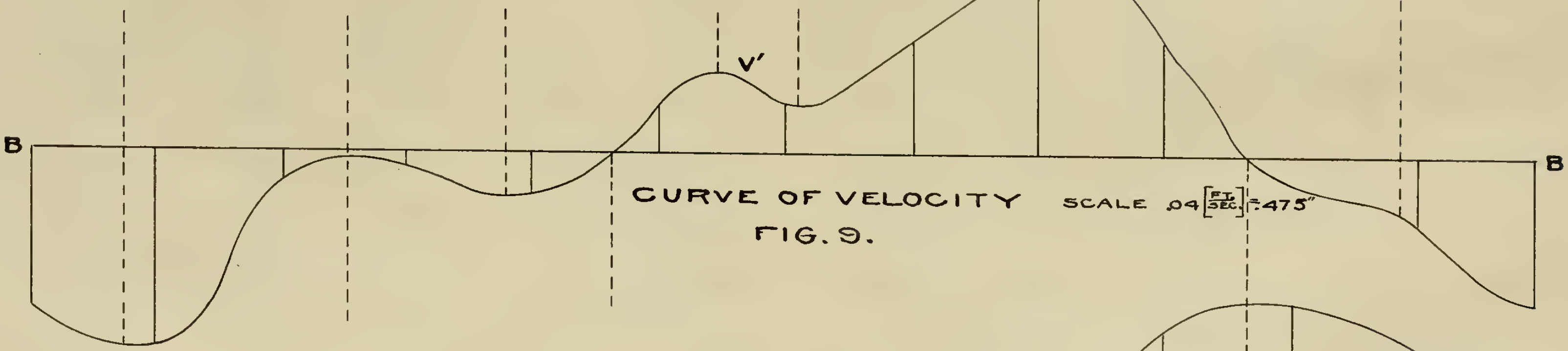
The equivalent weight of the revolving parts at crank radius (2.5 feet) equals 3,367,000 pounds. The velocity of the crank pin is $\frac{2.5 \times 2 \times 3.1416 \times 75}{60} = 19.63$ feet per second. As the number of revolutions per second equals $\frac{75}{60} = 1.25$, the number of spaces traversed per second equals 15, and the time for each space, .0667 second. The mass of the revolving parts equals,

$\frac{W}{g} = \frac{3,367,600}{32.2} = 104,584$. Hence $\frac{T}{M} = \frac{.06666}{104,584} = .0000006374$ equals acceleration for a force of one pound. Therefore $\frac{T}{M} \times F$ equals the velocity gained or lost during each interval, as shown in column A, Table IV.

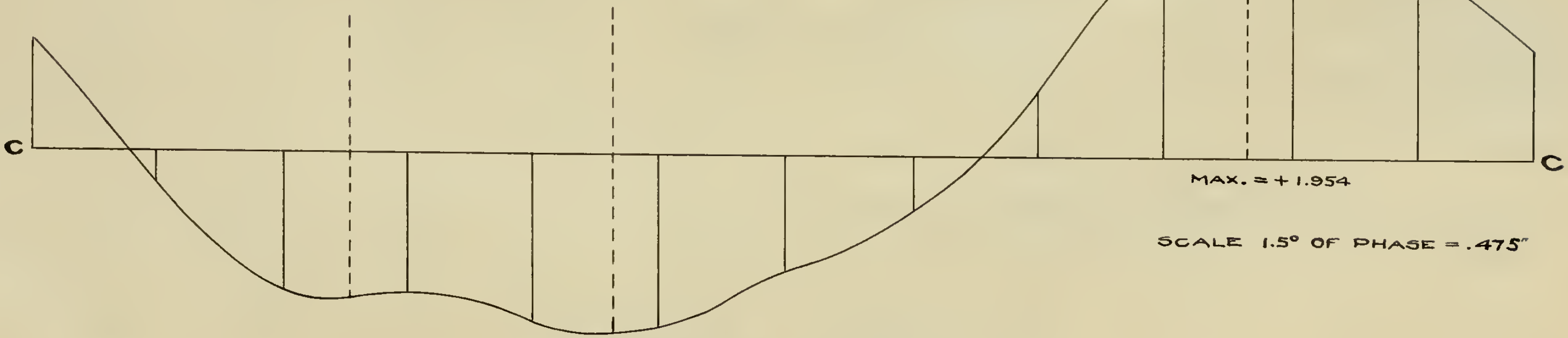
Now, if the velocity of the pin be assumed normal at the beginning of the stroke, the velocity attained up to the end of the various spaces will be equal to the algebraic sum of the velocities gained during each of the preceding spaces. These velocities attained up to the end of each space are shown in column V". As the actual velocity of the crank pin at the beginning of the stroke was not normal as assumed, it becomes necessary to correct the values of V" accordingly. The integrated sum of the velocities



CURVE OF EFFORT
FIG. 8.



CURVE OF VELOCITY
FIG. 9.



CURVE OF DISPLACEMENT
FIG. 10.

TABLE IV.

Space No.	F Tangential force in Lbs. above or below av. pressure. F= mean height of each space.	A Velocity gained during each 1/12 of a revolution. $A = \frac{1}{M} \times F$.	V" Velocity attained up to the end of divisions assuming 0 at beginning of stroke.	V' V" corrected to bring 0 to its proper place. V' = V" -.033.	V Average velocity during each interval.	S" Space actually passed over during each interval.	S' Displacement, assuming 0 at the beginning of stroke.	S S' corrected to bring 0 to its proper place. S = S' +.0018.	Deg. of arc. S x 22.92 1 foot = 22.92 degrees.	Deg. of Phase. There are 40 poles on the generator, consequently there are 20 changes of phase.
1	-13500	-.0086	-.0086	-.0416	-.0375	-.0025	-.00250	-.00060	-.0137	-0.274
2	+57000	+.0363	+.0277	-.0053	-.0261	-.00174	-.00424	-.00234	-.0536	-1.072
3	+ 4500	+.0029	+.0306	-.0024	-.0012	-.00008	-.00432	-.00242	-.0555	1.110
4	-10500	-.0067	+.0239	-.0091	-.0072	-.00048	-.00480	-.00290	-.0665	-1.330
5	+30000	+.0191	+.0430	+.0100	-.0015	-.00010	-.00490	-.00300	-.0688	-1.376
6	+ 1500	+.0009	+.0439	+.0109	+.0133	+.00089	-.00401	-.00211	-.0484	-0.968
7	+21000	+.0134	+.0573	+.0243	+.0155	+.00103	-.00298	-.00108	-.0247	-0.494
8	+30000	+.0191	+.0764	+.0434	+.0334	+.00223	-.00075	+.00115	+.0264	+0.528
9	-30000	-.0191	+.0573	+.0243	+.0390	+.00260	+.00185	+.00375	+.0859	+1.718
10	-48000	-.0306	+.0267	-.0063	+.0059	+.00039	+.00224	+.00414	+.0949	+1.898
11	-13500	-.0086	+.0181	-.0149	-.0097	-.00065	+.00159	+.00349	+.0800	+1.600
12	-28500	-.0181	-.0000	-.0330	-.0234	-.00156	+.00003	+.00193	+.0442	+0.884

above and below normal attained during one revolution must be zero, therefore the correction to be applied equals the algebraic sum of the velocities V" divided by the number of spaces (twelve in this case). Thus the correction is .033, and this amount must be deducted from the values of V" in order to arrive at the true velocity attained up to the end of the successive intervals. With these true velocities, given in column V', Table IV., as ordinates, plot the curve of velocity V', Fig.9 where BB represents the mean velocity of the pin.

From the curve of velocity, V', Fig.9 ascertain the average velocity above or below the mean velocity ,BB, during each space, shown in column V, Table IV. With these velocities given, the space actually passed over during each interval can be readily calculated by multiplying the valves of V by .0667, the time for one space. The figures in column S" were deduced in this way.

If the position of the pin be assumed normal at the beginning of the stroke, its distance from normal up to the end of the respective intervals will be equal to the algebraic sum of the spaces actually passed over, ahead of or behind the mean position, during each interval. Therefore, the figures in column S' are equal to the integrated sum of the preceding figures in column S". As the position of the crank-pin at the beginning of the stroke was not zero, as assumed, a correction must be applied to the values of S'. Since the integrated sum of the distances ahead of or behind the mean position must be equal to zero, the value of the correction is equal to the ratio of the algebraic sum of the values of S' to the number of spaces. The value of the correction is .00182, and is to be added to the values of S' to get the true displacement or distance from normal of the pin at the end of each

interval, the figures for the same being shown in column S. Since one foot corresponds to 22.92 degrees of arc, measured on the crank-pin circle, the number of degrees of arc from normal equals the product of the true distances in feet from normal (column S) by 22.92. The number of degrees of arc from normal deduced in this way are shown in the penultimate column of Table IV. Finally, as there are 40 poles on the generator, there will be 20 cycles or changes of phase per revolution, therefore one degree of arc equals 20 degrees of phase, and the displacement (shown in the last column of Table IV.) at the end of each interval may be calculated by multiplying the corresponding degrees of arc by 20. With the values of the displacement in degrees of phase from normal as ordinates, the curve of displacement, Fig.10, was plotted, in which CC represents the mean position of the crank pin.

To insure the satisfactory operation of two alternating current generators when working in parallel, the maximum amount of angular variation or displacement should not be allowed to exceed 2-1/2 degrees of phase departure from the mean position during any revolution, and as the maximum displacement shown by this final curve is well within the 2-1/2 degrees limit, the design of the unit in that respect may be considered satisfactory.